Neap:

Trial Examination 2007

VCE Specialist Mathematics Units 3 & 4

Written Examination 2

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 2 hours

Student's Name: ______________________________

Teacher's Name: ______________________________

Structure of Booklet

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	5	5	58
			Total 80

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator.

Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 22 pages with a detachable sheet of miscellaneous formulas in the centrefold.

Answer sheet for multiple-choice questions.

Instructions

Detach the formula sheet from the centre of this book during reading time.
Write **your name** and your **teacher's name** in the space provided above on this page.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2007 VCE Specialist Mathematics Units 3 & 4 Written Examination 2.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

An ellipse touches the line $x = 4$. It also touches the y-axis at (0, 1). If the height of the ellipse is 6 units, its equation is

A. $\frac{(x-2)^2}{16} + \frac{(y-1)^2}{36} = 1$

B. $\frac{(x+2)^2}{16} + \frac{(y+1)^2}{36} = 1$

C. $\frac{(x-2)^2}{4} + \frac{(y-1)^2}{36} = 1$

D. $\frac{(x-2)^2}{4} + \frac{(y-1)^2}{9} = 1$

E. $\frac{(x+2)^2}{4} + \frac{(y+1)^2}{9} = 1$

Question 2

A rational function is given by $f(x) = \frac{a + bx^2}{x^2}$.

The graph of $y = f(x)$

A. is symmetrical about the x-axis and has $y = a$ as its only straight line asymptote.
B. is symmetrical about the y-axis and has $x = 0$ as its only straight line asymptote.
C. is symmetrical about the x-axis and has $y = b$ as its only straight line asymptote.
D. is symmetrical about the x-axis and has $x = 0$ and $y = b$ as its straight line asymptotes.
E. is symmetrical about the y-axis and has $x = 0$ and $y = b$ as its straight line asymptotes.

 TEVSMU34EX2_QA_07.FM

Question 3

The complex number $z = \frac{4}{\sqrt{3}i - 1}$. The complex number $\bar{z}$ has modulus a and argument b.

The values of a and b are given by

A. $a = 2$ and $b = \frac{2\pi}{3}$.

B. $a = 2$ and $b = -\frac{2\pi}{3}$.

C. $a = 2$ and $b = \frac{\pi}{3}$.

D. $a = 4$ and $b = -\frac{\pi}{6}$.

E. $a = 4$ and $b = \frac{\pi}{6}$.

Question 4

Let the cube roots of $27i$ be represented by the complex numbers z_1, z_2 and z_3.

Which one of the following statements must be **false**?

A. On an Argand diagram, z_1, z_2 and z_3 are separated by an angle of $\frac{2\pi}{3}$ radians.

B. $z_1 = 3i$

C. $|z_2| = 3$

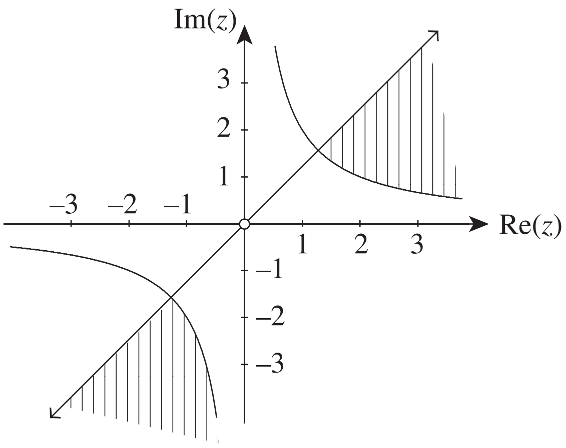
D. $(z_3)^{-1} = \frac{1}{3}\text{cis}\left(-\frac{5\pi}{6}\right)$

E. $z_1 + z_2 + z_3 = 0$

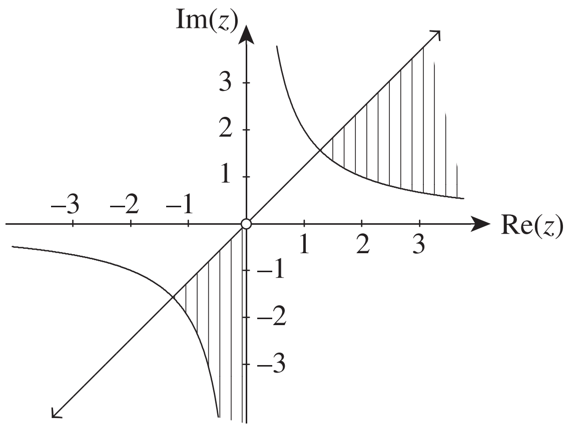
Question 5

The region of the complex plane defined by $\{z\text{: Im}(z^2) \geq 4\} \cap \left\{z\text{: } -\frac{3\pi}{4} \leq \text{Arg}(z) \leq \frac{\pi}{4}\right\}$ is represented by

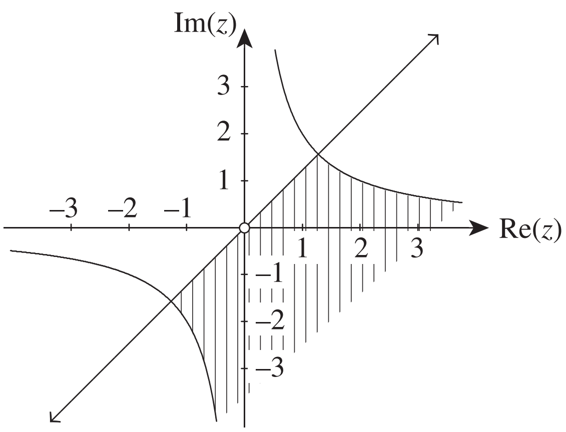
A.



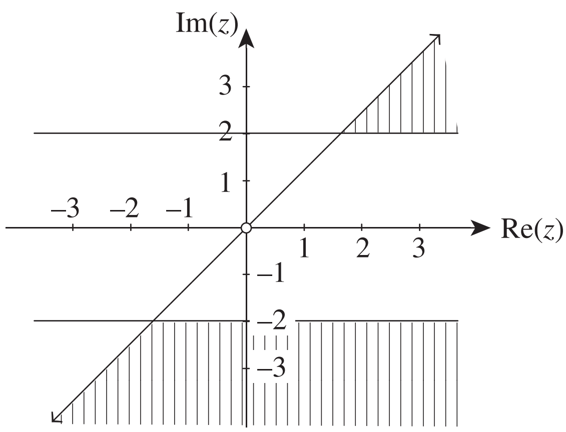
B.



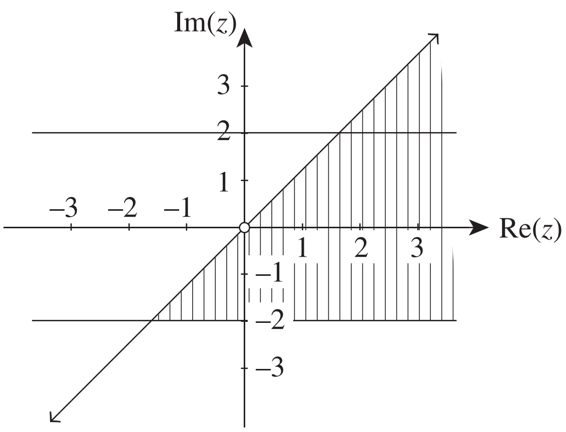
C.



D.



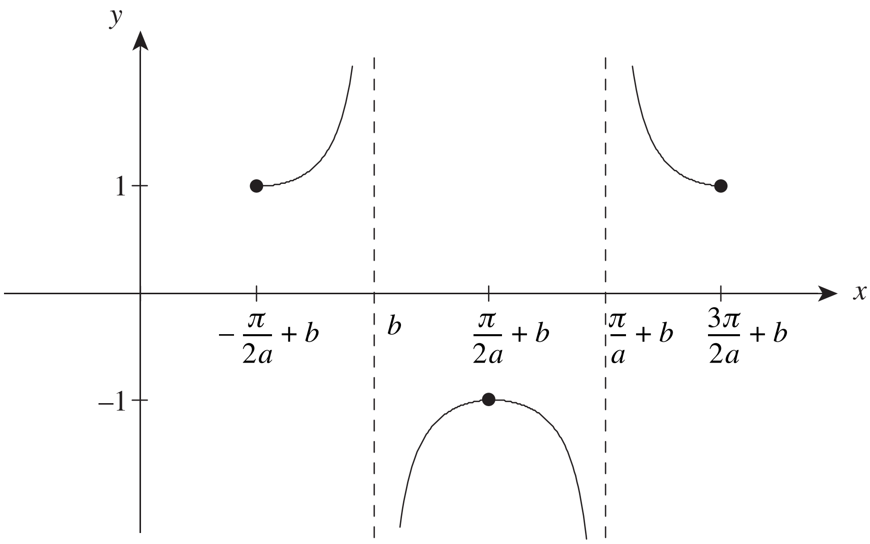
E.



 TEVSMU34EX2_QA_07.FM

Question 6

Part of the graph of $y = g(x)$ is shown below. The constants a and b are positive.



The rule for $g(x)$ could be

A. $g(x) = \sec(a(x - b))$

B. $g(x) = -\sec(a(x - b))$

C. $g(x) = \text{cosec}(a(x - b))$

D. $g(x) = \text{cosec}(a(b - x))$

E. $g(x) = \text{cosec}(b(x - a))$

Question 7

The domain of $f(x) = \sin(2x)$ is chosen so that it is a one-to-one function. The domain and range of the inverse function, $f^{-1}(x)$, could be, respectively,

A. $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ and $[1, -1]$.

B. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $[1, -1]$.

C. $[-1, 1]$ and $\left[\frac{3\pi}{4}, \frac{5\pi}{4}\right]$.

D. $[-1, 1]$ and $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

E. $[-1, 1]$ and $[-\pi, \pi]$.

Question 8

The vector resolute of $2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k}$ in the direction of $3\underset{\sim}{i} - 2\underset{\sim}{k}$ is

A. $\frac{5}{13}(3\underset{\sim}{i} - 2\underset{\sim}{k})$

B. $\frac{5}{14}(2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k})$

C. $\frac{8}{13}(3\underset{\sim}{i} - 2\underset{\sim}{k})$

D. $\frac{8}{14}(2\underset{\sim}{i} - 3\underset{\sim}{j} - \underset{\sim}{k})$

E. $\frac{11}{13}(3\underset{\sim}{i} - 2\underset{\sim}{k})$

Question 9

The equation of the tangent to the curve $y = \cos^{-1}\left(\frac{x}{2}\right)$ at the point where $x = 1$ is

A. $y - \frac{\pi}{6} = -\frac{1}{\sqrt{3}}(x - 1)$

B. $y - \frac{\pi}{6} = \frac{1}{\sqrt{3}}(x - 1)$

C. $y - \frac{\pi}{3} = -\frac{1}{\sqrt{3}}(x - 1)$

D. $y - \frac{\pi}{3} = \frac{1}{\sqrt{3}}(x - 1)$

E. $y - \frac{\pi}{3} = -\frac{1}{2\sqrt{3}}(x - 1)$

Question 10

With a suitable substitution, $\int_0^{\frac{\pi}{4}} \sqrt{1-\sin(2x)}(1-2\cos^2(x))\ dx$ can be expressed as

A. $\frac{1}{2}\int_0^{\frac{\pi}{4}} u^{\frac{1}{2}}\ du$

B. $\frac{1}{2}\int_0^{\frac{\pi}{4}} (u-1)^{\frac{1}{2}}\ du$

C. $\frac{1}{2}\int_0^{1} u^{\frac{1}{2}}\ du$

D. $\frac{1}{2}\int_1^{0} (u-1)^{\frac{1}{2}}\ du$

E. $\frac{1}{2}\int_1^{0} u^{\frac{1}{2}}\ du$

Question 11

With a suitable substitution, $\int_0^{\frac{\pi}{6}} \sin^3(6x)dx$ can be expressed as

A. $\frac{1}{6}\int_0^{\frac{\pi}{6}} (1-u^2)du$

B. $\frac{1}{6}\int_0^{\frac{\pi}{6}} (u^2-1)du$

C. $\frac{1}{6}\int_1^{-1} (1-u^2)du$

D. $\frac{1}{6}\int_1^{-1} (u^2-1)du$

E. $6\int_1^{-1} (1-u^2)du$

Question 12

To solve the differential equation $\frac{dy}{dx} = \cos^3(x)$, with the initial condition $y = 1$ when $x = 0$, Euler's method is used with a step size of 0.1.

When $x = 0.2$, the approximation obtained for y is given by

A. $1 + 0.1\cos^3(0.1)$

B. $1 + 0.2\cos^3(0.2)$

C. $1.1 + 0.1\cos^3(0.1)$

D. $1 + 0.1\cos^3(0.1) + 0.1\cos^3(0.1)$

E. $1.1 + 0.2\cos^3(0.1)$

Question 13

A particle of mass 10 kg is projected vertically upwards from O with a velocity of u m/s. The particle experiences a resistive force of $\frac{v^2}{10}$ newtons, where v m/s is the velocity of the particle at any time, t.

$\frac{dv}{dt}$ is equal to

A. $\frac{100g - v^2}{100}$

B. $\frac{v^2 - 100g}{100}$

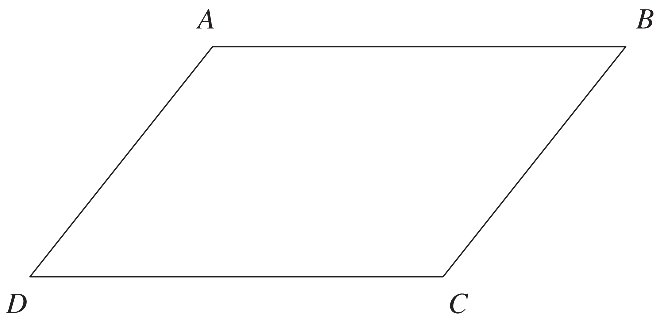
C. $-\frac{10g + v^2}{10}$

D. $-\frac{100g + v^2}{100}$

E. $\frac{100g + v^2}{100}$

 TEVSMU34EX2_QA_07.FM

Question 14



To prove that *ABCD* is a rhombus, it is sufficient to show that

A. $\overrightarrow{AB} = \overrightarrow{DC}$ and $|\overrightarrow{AB}| = |\overrightarrow{AD}|$

B. $\overrightarrow{AB} = \overrightarrow{DC}$ and $\overrightarrow{AD} = \overrightarrow{BC}$

C. $\overrightarrow{AB}.\overrightarrow{AD} = 0$

D. $\overrightarrow{AC}.\overrightarrow{BD} = 0$

E. $\overrightarrow{AB} = \overrightarrow{DC}$

Question 15

A particle moves in the x–y plane with a velocity vector given by $\dot{\underset{\sim}{r}}(t) = e^{-t}\underset{\sim}{i} - 3e^{-3t}\underset{\sim}{j}$ for $t \geq 0$. At $t = 0$, the particle is at the origin, i.e. $\underset{\sim}{r} = \underset{\sim}{0}$.

$\underset{\sim}{r}(t)$ is equal to

A. $(1 - e^{-t})\underset{\sim}{i} + (e^{-3t} - 1)\underset{\sim}{j}$

B. $-e^{-t}\underset{\sim}{i} + 9e^{-3t}\underset{\sim}{j}$

C. $-e^{-t}\underset{\sim}{i} + e^{-3t}\underset{\sim}{j}$

D. $(e^{-t} - 1)\underset{\sim}{i} + (1 - e^{-3t})\underset{\sim}{j}$

E. $-(1 + e^{-t})\underset{\sim}{i} + (1 - e^{-3t})\underset{\sim}{j}$

Question 16

The cosine of the angle between $\underset{\sim}{a} = \underset{\sim}{i} - 2\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ is

A. 0

B. $\frac{2}{3\sqrt{5}}$

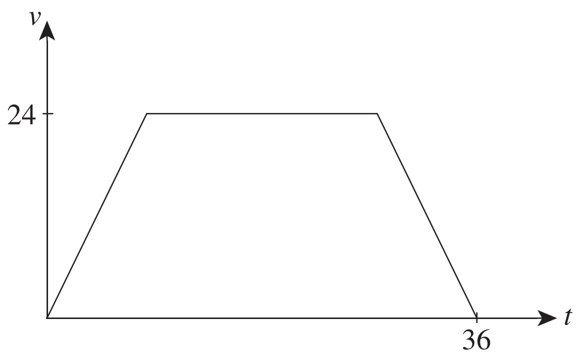
C. $-\frac{2}{3\sqrt{5}}$

D. $-\frac{2}{3\sqrt{3}}$

E. $\frac{2}{\sqrt{5}}$

Question 17

The velocity–time graph for a particle is shown below.



If the particle travelled a total distance of 600 m, how far did the particle travel while moving at a constant velocity?

A. 264 m
B. 336 m
C. 300 m
D. 432 m
E. 504 m

Question 18

A body of mass 8 kg is travelling in a straight line. Its velocity decreases from 7 m/s to 1 m/s in 3 seconds. The change in momentum of the particle in kg m/s, in the direction of its motion, is

A. –56
B. –48
C. 8
D. 16
E. 48

Question 19

A block, m kg, is projected directly down a rough inclined plane with an initial speed of 5 m/s. The plane is inclined at an angle θ where $\sin(\theta) = \frac{5}{13}$. The block comes to rest after travelling 8 metres down the plane, and the coefficient of friction is μ.

The acceleration (in m/s^2) of the particle down the plane is

A. $\frac{g}{13}(12 - 5\mu)$
B. $\frac{g}{13}(12\mu + 5)$
C. 0
D. $\frac{5g}{13}$
E. $\frac{g}{13}(5 - 12\mu)$

Question 20

At time t seconds, the position of a body of mass 2 kg is given by $\underset{\sim}{r} = -\sqrt{3}\cos(t)\underset{\sim}{i} + \sin(t)\underset{\sim}{j}$.

The resultant force in newtons acting on the particle at any time t has a magnitude of

A. $\sqrt{12\cos^2(t) + 4\sin^2(t)}$

B. $\sqrt{3\sin^2(t) + \cos^2(t)}$

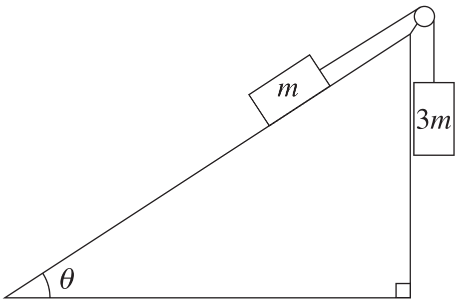
C. $\sqrt{12\sin^2(t) + 4\cos^2(t)}$

D. $\sqrt{3\cos^2(t) + \sin^2(t)}$

E. $2\sqrt{3}\cos(t)\underset{\sim}{i} - 2\sin(t)\underset{\sim}{j}$

Question 21

Two particles of mass m kg and $3m$ kg are connected by a light string that passes over a smooth pulley as shown in the diagram below. The particle of mass m rests on a rough inclined plane.



If the particle of mass m kg is on the point of moving up the plane, the coefficient of friction, μ, is

A. $\dfrac{\sin(\theta) - 3}{\cos(\theta)}$

B. $\dfrac{3 - \sin(\theta)}{\cos(\theta)}$

C. $\dfrac{\cos(\theta)}{3 - \sin(\theta)}$

D. $3 - \sin(\theta)$

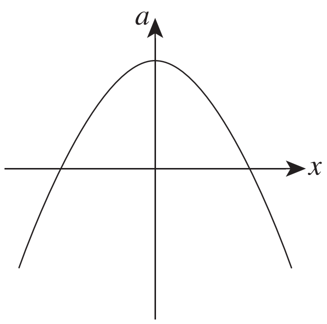
E. $\tan(\theta)$

Question 22

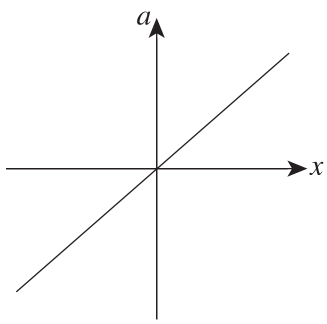
A body is moving along a straight line in such a way that its velocity, v m/s, can, at any time, be expressed in terms of its displacement, x m, from the origin by the equation $v^2 = 12 - 4x^2$.

Which one of the following graphs best describes the relationship between acceleration and displacement for the body's motion?

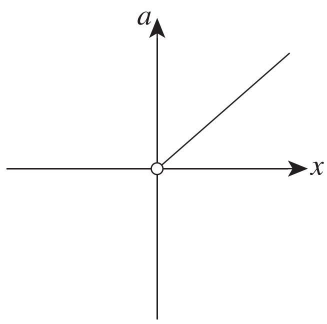
A.



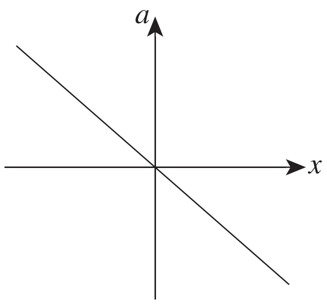
B.



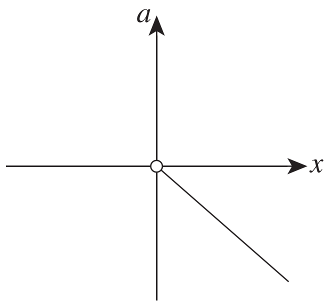
C.



D.



E.



Neap:

Trial Examination 2007

VCE Specialist Mathematics Units 3 & 4

Written Examination 2

Formula Sheet

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

SPECIALIST MATHEMATICS FORMULAS

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$ $\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$ $\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$ $\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

 TEVSMU34EX2_FS_07.FM

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\text{cis}\theta$

$|z| = \sqrt{x^2 + y^2} = r$

$-\pi < \text{Arg}(z) \le \pi$

$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$

$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}(x^n) = nx^{n-1}$

$\displaystyle\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \; n \ne -1$

$\dfrac{d}{dx}(e^{ax}) = ae^{ax}$

$\displaystyle\int e^{ax} dx = \frac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}(\log_e(x)) = \dfrac{1}{x}$

$\displaystyle\int \frac{1}{x} dx = \log_e|x| + c, \text{ for } x > 0$

$\dfrac{d}{dx}(\sin(ax)) = a\cos(ax)$

$\displaystyle\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}(\cos(ax)) = -a\sin(ax)$

$\displaystyle\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}(\tan(ax)) = a\sec^2(ax)$

$\displaystyle\int \sec^2(ax)dx = \frac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}(\sin^{-1}(x)) = \dfrac{1}{\sqrt{1-x^2}}$

$\displaystyle\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \; a > 0$

$\dfrac{d}{dx}(\cos^{-1}(x)) = \dfrac{-1}{\sqrt{1-x^2}}$

$\displaystyle\int \frac{-1}{\sqrt{a^2 - x^2}}dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \; a > 0$

$\dfrac{d}{dx}(\tan^{-1}(x)) = \dfrac{1}{1+x^2}$

$\displaystyle\int \frac{a}{a^2 + x^2}dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \dfrac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \dfrac{1}{2}(u + v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum:	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion:	$\underset{\sim}{R} = m\underset{\sim}{a}$
sliding friction:	$F \le \mu N$

END OF FORMULA SHEET

SECTION 2

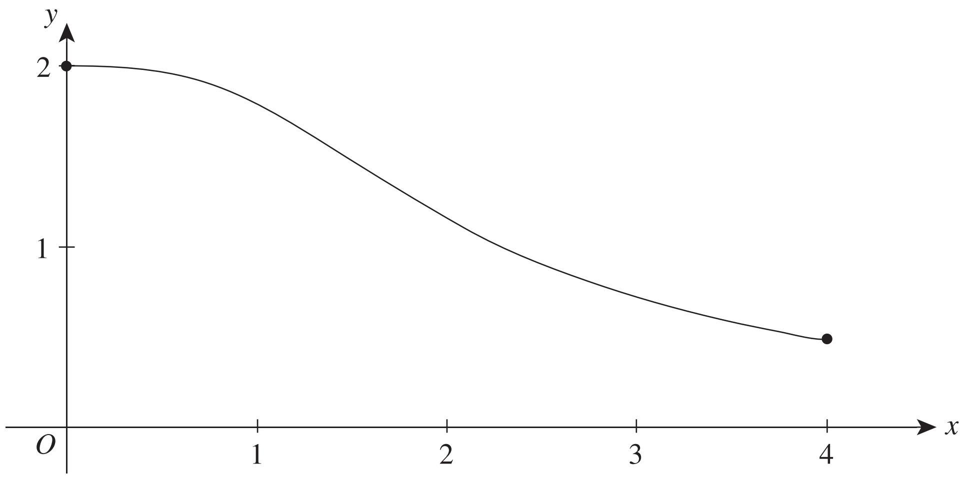
Instructions for Section 2

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

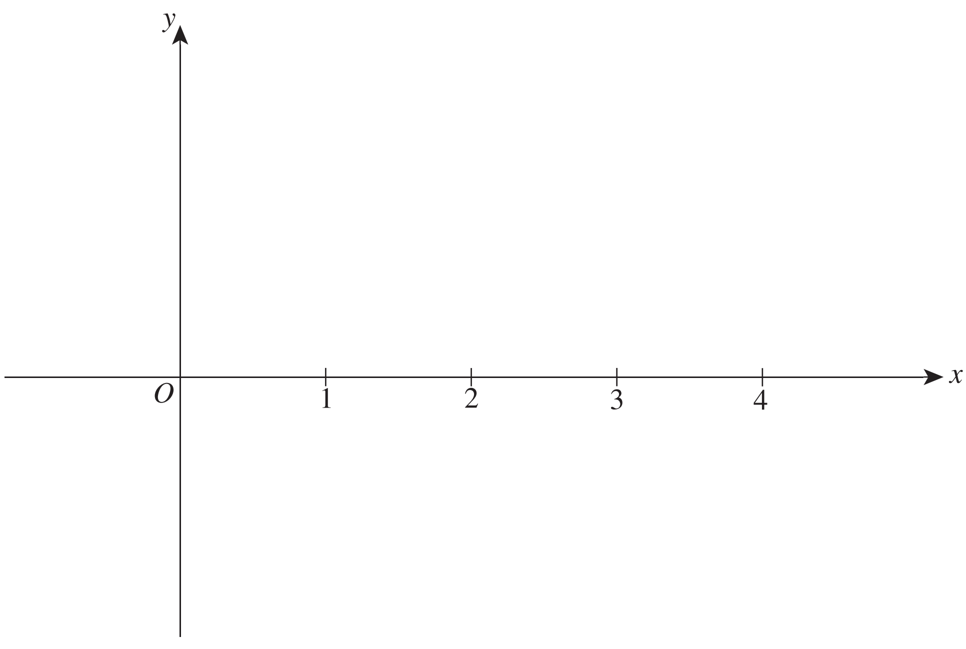
Question 1

Consider the function $f(x) = \dfrac{4}{\sqrt{x^3 + 4}}$ for $0 \le x \le 4$.

A sketch of $y = f(x)$ is shown on the set of axes below.



a. Sketch a graph of $y = f'(x)$ on the set of axes below. Indicate a scale on the y-axis.



2 marks

b. Find the coordinates of the point where the maximum rate of decrease for *f* occurs, correct to two decimal places.

2 marks

c. Find the maximum rate of decrease of *f*, correct to two decimal places.

1 mark

d. The area enclosed by the graph of *f*, the line $x = 0$ and $x = a$ is rotated 360° about the *x*-axis to form a solid of revolution.

If the volume of the solid is 15 cubic units, find the value of *a*. Express your answer correct to two decimal places.

2 marks

Total 7 marks

Question 2

Consider the vectors $\underset{\sim}{a} = \underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$, $\underset{\sim}{b} = 2\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{c} = 4\underset{\sim}{i} + p\underset{\sim}{j} + 5\underset{\sim}{k}$ where *p* is a real constant.

a. Find the unit vector in the direction of $\underset{\sim}{a}$.

1 mark

b. Suppose that $p = -1$.

i. Verify that $\underset{\sim}{b}$ is perpendicular to $\underset{\sim}{c}$.

ii. A unit vector perpendicular to both $\underset{\sim}{b}$ and $\underset{\sim}{c}$ is given by $\hat{\underset{\sim}{n}} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$ where $x < 0$.

Find $\hat{\underset{\sim}{n}}$ and **hence** describe the geometrical relationship between $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$.

iii. The points A, B and C have position vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ respectively relative to an origin O. The points O, A, B and C are four of the eight vertices of a cuboid.
Find the volume of the cuboid.

1 + 4 + 2 = 7 marks

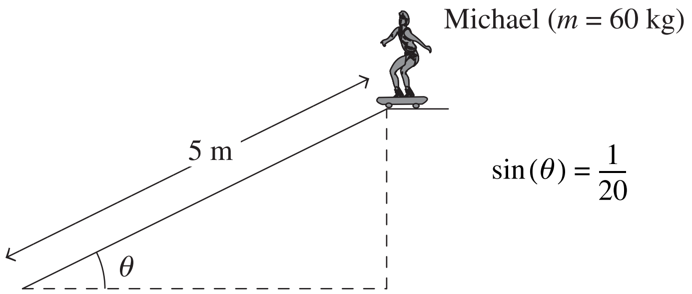
c. Now suppose that $p \neq -1$.

If vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are linearly dependent, find the value of p.

4 marks
Total 12 marks

Question 3

A teenage skateboarder, Michael, of mass 60 kg is about to go down a smooth 1-in-20 slope $\left(\sin(\theta) = \frac{1}{20}\right)$ which is 5 metres long.



a. Find Michael's acceleration down the slope, correct to the nearest 0.01 m/s^2.

3 marks

b. Using your final answer from **a.**, determine Michael's speed at the bottom of the slope, correct to the nearest 0.01 m/s.

2 marks

c. How many seconds does Michael take to reach the bottom of the slope, correct to the nearest second?

2 marks

d. When Michael reaches the bottom of the slope, he is on a level but bumpy piece of ground on which he experiences a retarding force which is inversely proportional to the square of his speed. After travelling 5 metres on the level ground, his speed is reduced to 1.5 m/s.

i. After how many metres of travel on the level ground does Michael reach a speed of 1 m/s, correct to the nearest metre?

ii. After how many seconds of travel on the level ground does Michael reach this speed of 1 m/s, correct to the nearest second?

5 + 3 = 8 marks
Total 15 marks

Question 4

A particle moves in the *x–y* plane so that after *t* seconds its velocity is given by

$\dot{\underset{\sim}{r}}(t) = (9t - 3t^2)\underset{\sim}{i} + \log_e(1 + (t-3)^4)\underset{\sim}{j}$ m/s, $t \geq 0$.

a. Find the speed of the particle when $t = 1$, correct to two decimal places.

2 marks

b. Find when the particle is at rest.

3 marks

c. Find the gradient of the curve along which the particle moves when $t = 1$, correct to two decimal places.

2 marks

d. When $t = 1$, the particle is at point *P*.

Given that $\underset{\sim}{r}(0) = -10\underset{\sim}{i} + 2\underset{\sim}{j}$, find the *y*-coordinate of *P*, correct to two decimal places.

3 marks
Total 10 marks

Question 5

Let $z = \text{cis}\left(\frac{\pi}{6}\right)$.

a. Use de Moivre's theorem to express the following in Cartesian form.

i. z^2

ii. z^4

2 + 1 = 3 marks

b. Show that $z^4 - z^2 + 1 = 0$.

2 marks

c. **i.** Deduce two linear factors, in Cartesian form, of the polynomial $P(z) = z^4 - z^2 + 1$.

2 marks

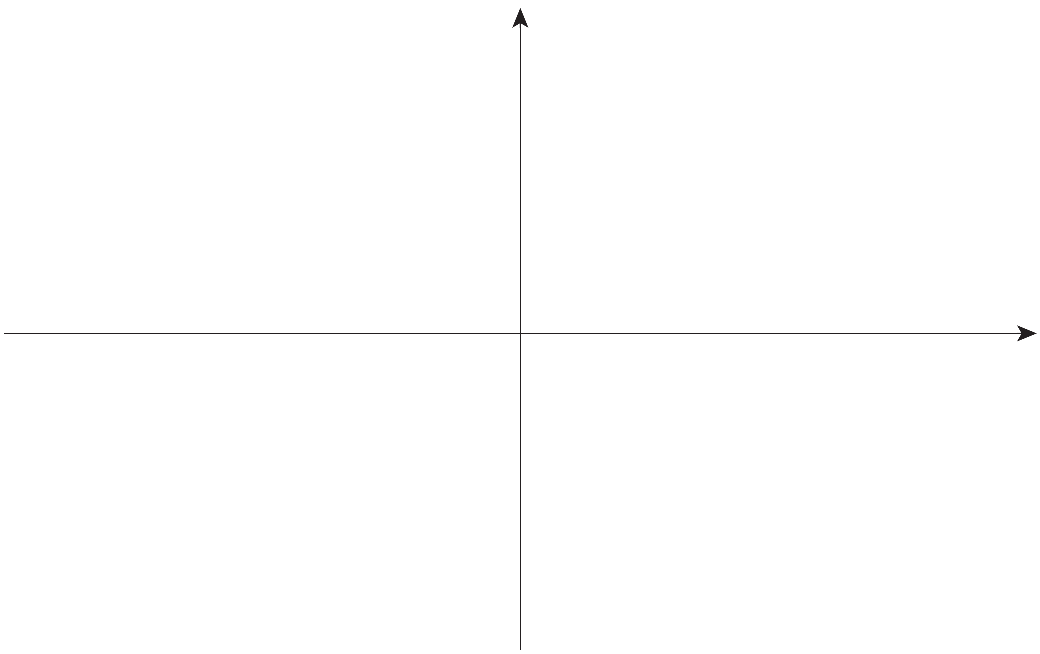
ii. Write down the product of the two factors from **c. i.** in expanded, simplified form.

1 mark

d. By first writing $P(z)$ in the form $(z^2 + 1)^2 - (az)^2$, where a is a positive real number, and using your answers to **c.** or another method, find in Cartesian form the other linear factors of $P(z)$.

3 marks

e. **i.** Using the axes below, show the solutions of the equation $z^4 - z^2 + 1 = 0$ on a clearly labelled Argand diagram.



ii. On the diagram in **e. i.**, shade the region of the complex plane given by

$\{z: |z| \leq 1\} \cap \left\{z: \text{Im}(z) > \frac{1}{2}\right\}$.

1 + 2 = 3 marks
Total 14 marks

END OF QUESTION AND ANSWER BOOKLET

 TEVSMU34EX2_QA_07.FM